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Development of a Finite Element Computer Simulation Platform for a Coupled Reservoir and Geomechanics System

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Abstract

The aim of this article is to describe the development of a finite element computer platform for the modeling, analysis, and simulation of a coupled reservoir and geomechanics system. The conditions that are assumed are the simplest possible, so that we assume homogeneity and isotropy in the material, an elastic behavior of the rock with small deformations, a quasi-static equilibrium, the absence of fractures and isothermal state. The study evolves sequentially in stages, from the flow of a single fluid in a porous medium, through the two-phase case, up to the three-phase black oil model. The starting point is based on a mathematical model that is an extension of Biot's theory for flow in porous media, and which accounts for coupling the reservoir of fluid flow to the rock deformation and stress states. The mathematical model is reformulated in variational terms, in order to get a proper formulation that enables us to apply the weighted residuals methods of Galerkin with linear interpolation functions. The use of linear interpolant functions reduces the computational work but at the cost of having to face some inestabilities in the solutions. Once the model is discretized, we make use of appropriate software to built an unstructured meshing domain using tetrahedral elements. We carried out a computational model founded in the design of a program based in the C++ language, to perform the tasks of numerical calculations of the coefficients, assembling the stiffness element by element matrices, and for solving the resulting equations. It is intended to develop the computer program in a modular structure, so that the solution of the coupled geomechanics/reservoir system can be performed through an iterative procedure. The development of this code is still in its initial stage, but the approachs show promise. At the end two case studies are presented, which are compared with analytical solution for verification of the approach

Introduction

When a stress sensitive hydrocarbon reservoir is subject to exhaustion of the pressure during the production process, or is subject to an increase in pressure due to the injection of fluid, the rock skeleton and overburden undergo anisotropic changes in states stress. This leads to significant deformation and changes in physical properties of porosity, compressibility, and permeability. In turn, these changes affect the conditions of fluid flow that may be reflected in the efficiency of hydrocarbon recovery, lead to

problems of wellbore stability, and cause surface subsidence and compaction, despite that the phenomenon of compaction in many cases acts as a driving mechanism.

The simultaneous interaction of the mechanisms of changes in pressure followed by changes in the state of in situ stress and deformation of rock, and the resulting changes in volume, porosity and permeability, yield to the interconnection of two different physical processes, Gutierrez, M., Lewis, R. and Masters, I, (2001): (i) the movement of fluids through the pores, and (ii) rock deformation.

Consequently, in order to incorporate the changes in the dependent physical properties of stress changes in reservoir modeling, it is necessary to perform a coupling of the rock deformation governing equations and the fluid flow governing equations. In conventional simulators this interaction is not considered in its entirety, in fact the porosity is determined by a simple formula depending on the pressure, and for pore volume changes a compressibility factor is applied and adjusted at each iteration level, but giving only partially account of such changes.

In general it is not possible to find analytical solutions to problems of single or multiphasic nonlinear fluid flow, coupled with geomechanics. However there is a range of numerical methods available for application and for obtaining approximate solutions. Using an appropriate numerical method, with precision, convergence and stability properties, which in turn leads to the implementation of more economical and efficient computational methods, is in itself a challenge that is faced to the investigator.

This research aims to develop a numerical and computational platform based on the finite element method and C++ language for modeling, analysis, and simulation of stress sensitive hydrocarbon reservoirs. The development of this code is still in its initial stage, but the approachs show promise.

The physical imposed conditions are as simple as possible, so that the homogeneity and isotropy, elastic behavior of the rock with small strains, quasi-static equilibrium, no fractures, and isothermal state is assumed. The study evolves sequentially in stages, from the flow of a single fluid in a porous medium, through the two-phase case, up to the three-phase black oil model.

Theoretical and Conceptual Bases

The Geomechanics Physics and Mathematical Model

The founding mathematical model can be viewed as an extension of classical Biot theory for flow in porous media, which considers the flow of fluids in porous media as a tightly coupled system with environmental geomechanics.

The hypothesis that complement the physical model are: (1) it is assumed that the solid phase comprises a porous particle skeleton saturated by one or more fluids (liquid or gaseous hydrocarbons and water); (2) the shear stresses in the fluid phases are small, while on the solid phase a surrounding pressure is exerted; (3) the solid material is isotropic with respect to the mechanical properties.

Additionally we also assume: (4) the equilibrium conditions express only the quasi-static behavior, and do not reflect inertial effects; (5) the magnitude of the deformations are considered small enough so that the linear elastic theory is applicable; and (6) it is assumed that the dynamic process is isothermal.

The three basic principles on which this model is based, Chen, H., Teufel, L. and Lees, R. (1995), are: equilibrium condition of stress, strain-displacement relationship and the stress-strain constitutive equation.

Equilibrium equation:

The equilibrium equation accounts by the balance of forces on the body:

$$\nabla \cdot \sigma + F = 0 \tag{1}$$

where σ is the stress tensor with components $\sigma_{i,\ j}$, and is the vector of forces on the body with F_{i} components.

By denoting with σ' the efective stress, and with p the fluid pressure, the vectorial equation (1) can be rewritten as:

$$\nabla \cdot (\sigma' - \alpha p I) + F = 0 \tag{2}$$

where α is the Biot constant, and I is the identity matrix.

Strain/displacement relationship given by means:

$$\varepsilon = \frac{[\nabla u + (\nabla u)^T]}{2} \tag{3}$$

where ε is the strain tensor with components $\varepsilon_{ij,\ u}$, is the displacement vector, with components $u_{j,}\ j = 123$, and $\triangledown\ u$ is the Jacobian matrix of u.

Stress/strain constitutive equation given by:

$$\sigma = 2\mu\varepsilon + \lambda(\nabla \cdot u)I - \alpha p I = \sigma' - \alpha p I \tag{4}$$

where σ is the stress tensor, ε is the strain tensor, $\sigma' = 2\mu\varepsilon + \lambda(\triangledown \cdot u)I$ is effective stress, λ and μ are Lamé elastic constants, and $\triangledown \cdot u$ denotes the divergence of the vector u, and this term accounts for the pore volume deformation.

By inserting expressions (5) and (4) in equation (3), and after certain algebra, we finally obtain the equation for the geomechanics model:

$$\mu\nabla^2 u + (\lambda + \mu)\nabla(\nabla \cdot \mathrm{u}) = -F + \alpha\nabla \mathrm{p} \tag{5}$$

which are the Navier-Stokes equation for a saturated porous medium.

The Single Fluid Flow Mathematical Model

The two basic principles for fluid flow in a porous medium are mass conservation, Darcy's law of velocity, and the variables of interest are fluid content η, the fluid flux v_f, and the volumetric fluid source^s.

Mass conservation can be given as:

$$\frac{\partial \eta}{\partial t} = \nabla \cdot v_f + s \tag{6}$$

Darcy's law of velocities can be expressed as:

$$v_f = -\frac{K}{\mu}\nabla(p - \rho_f g z) \tag{7}$$

where p is the pore pressure, K is the permeability tensor, that we assume as diagonal, μ is the fluid viscosity, g is the gravity constant, and z is the depth.

It is natural to assume that the fluid content η depend on the fluid pressure and the volumetric change $\triangledown \cdot u$, more specifically

$$\eta = \alpha\nabla \cdot u + \frac{1}{M}p \tag{8}$$

where α is the Biot constant, M is the Biot module, $\frac{1}{M}p$ measures the amount of fluid that can be injected on a fixed material volume, and $\alpha\triangledown \cdot u$ measures the amount of fluid that can be squeezed out.

For the case of a fully connected pore space and a microscopically homogeneous and isotropic material, Detournay and Cheng (1993) show that α and M are related to the porosity ϕ, the drained skeleton bulk modulus K_s, and the fluid bulk modulus K_f, as follow:

$$\alpha = 1 - \frac{K_s}{K_f} \tag{9}$$

$$\frac{1}{M} = \frac{\phi}{K_f} + \frac{\alpha - \phi}{K_s} \tag{10}$$

By substituting expressions (7) and (8) into the mass conservation equation we find the equation for the mathematical model of the fluid flow:

$$\alpha\nabla \cdot \dot{u} + \frac{1}{M}\dot{p} - \nabla \cdot \left(\frac{K}{\mu}\nabla(p - \rho_f g z)\right) = s \tag{11}$$

Strong Formulation of the Problem

Let Ω be the region of interest, and ∂Ω its boundary, with outward normal vector *v*, and T the time interval considered, and decompose the boundary of Ω in the form

$$\partial\Omega = \Gamma_u \cup \Gamma_t = \Gamma_p \cup \Gamma_f \tag{12}$$

where Γ_u, Γ_t, Γ_p *and* Γ_f are the displacement boundary, the traction boundary, the pressure boundary, and the flux boundary, respectively, with

$$\Gamma_u \cap \Gamma_t = \Gamma_p \cap \Gamma_f = \emptyset \tag{13}$$

The strong boundary initial value problem for the fluid-flow/geomechanics systems can be established as:

Find functions u(x, t) and p(x, t) with continuous second derivatives on ΩxT, such that satisfy the equations

$$(\lambda+\mu)\nabla(\nabla\cdot u) + \mu\nabla^2 u - \alpha\nabla p + F = 0 \tag{14}$$

$$\alpha(\nabla\cdot\dot{u}) + \frac{1}{M}\dot{p} - \nabla\cdot\left(\frac{k}{\mu}\nabla\left(p-\rho_f g z\right)\right) = s \tag{15}$$

with boundary conditions:

$p(x,t) = \bar{p}\ on\ \Gamma_p, \quad -\frac{K}{\mu}(\nabla p - \rho_f g \nabla Z)\cdot v = \bar{q}\ on\ \Gamma_f, \quad u(x,t) = u_D\ on\ \Gamma_u, \quad \sigma'\cdot v = \bar{t}\ on\ \Gamma_t$

and initial conditions:

$p(x, 0) = p_o$, $u(x, 0) = u_o$.

In general it is not possible to find analytical solutions for the problem formulated under this perspective. However, the strong approach of the problem can be reformulated using a variational approach, in order to obtain a formulation that enables us to find approximate solutions by means of a suitable numerical method, which in our case will be the weighted residual Galerkin method, with linear interpolation functions.

Multiphase Coupled Geomechanics and Reservoir Flow Model Flow

The mathematical description for coupling porous flow and geomechanics must account for the geomechanical interactions between the reservoir and its surrounding environment (overburden, underburden and sideburden), in order to provide a realistic modeling of the actual geomechanical boundary conditions, Osorio, J., Chen, H. and Teufel, L. (1999). The surrounding domain needs to be extensive enough to ensure that its boundaries are not perturbed by reservoir production or injection during the time period of interest.

The coupling process is characterized by the interaction of essentially two different physical phenomena within overlapping material domains. Although we have assumed the two domains to be the same, solving displacements takes much more CPU time than for pressure and concentrations, thus assuming a large domain for the flow calculation will not induce substantial computational overhead.

Black Oil Model

The black oil model, Aiz, K. and Settari, A., (1979), uses a simple PVT description in which the hydrocarbon chemical species are lumped together to form two components at surface conditions: a heavy hydrocarbon component called “oil”, and a light hydrocarbon component called “gas”, for which the chemical composition remains constant for all times. At reservoir conditions, the gas component may be partially or totally dissolved in the oil phase, forming one or two phases (liquid and gas) that do not dissolve in the water phase.

The black oil model is often formulated as conservation of volumes at standard conditions rather than conservation of masses, by introducing volume factors $B_\beta = \frac{V_\beta}{V_{\beta s}}$ (V_β and $V_{\beta s}$ are volumes occupied by a bulk of component $\beta = w, o, g$ at reservoir and standard conditions), and a gas solubility factor $R_{so} = \frac{V_{gs}}{V_{os}}$, , which

is the volume of gas, measured at standard conditions, dissolved at reservoir conditions in a unit of stock tank (at surface conditions).

There are several options to choose primary variables for the black oil model, one of them, Gai, X., (2004) is to choose water phase pressure p_w, water concentration N_w, and gas concentration N_g. Pore pressure is taken to be the wetting phase pressure, i.e., water pressure. Some authors, as Lewis., R. and Schrefler, B. (1998) adopt a pressure averaged pore expression as primary variable.

The resulting conservation laws with incorporated geomechanics effects reads:

Conservation equations:

$$\begin{aligned}
&\frac{\partial}{\partial t}\left[N_w\left(\phi^0+\alpha(\nabla\bullet \mathrm{u}-\nabla\bullet \mathrm{u}_0)+\tfrac{1}{M}(p-p_0)\right)\right]+\nabla\bullet v_w=q_w\\
&\frac{\partial}{\partial t}\left[N_o\left(\phi^0+\alpha(\nabla\bullet \mathrm{u}-\nabla\bullet \mathrm{u}_0)+\tfrac{1}{M}(p-p_0)\right)\right]+\nabla\bullet v_o=q_o\\
&\frac{\partial}{\partial t}\left[(N_g+R_{so}N_o)\left(\phi^0+\alpha(\nabla\bullet \mathrm{u}-\nabla\bullet \mathrm{u}_0)+\tfrac{1}{M}(p-p_0)\right)\right]+\nabla\bullet v_g=q_g
\end{aligned} \tag{16}$$

where $N_w=\frac{S_g}{B_g}, N_o=\frac{S_o}{B_o}$, *and* $N_g=\frac{S_g}{B_g}+R_{so}\frac{S_o}{B_o}$ stand for water, oil and gas concentrations, respectively, S_β, $\beta = w, o, g$ denotes saturations for water, oil and gas saturations, and is the chosen primary pressure.

The repeatedly inserted bracket term $\phi^*\equiv\phi^0+\alpha(\nabla\bullet \mathrm{u}-\nabla\bullet \mathrm{u}_0)+\frac{1}{M}(p-p_0)$ accounts by the relative porosity ϕ^* which is introduced as a coupling parameter in the process.

Darcy velocities:

$$\begin{aligned}
v_w&=\frac{Kk_{rw}}{\mu_{fw}B_w}(\nabla p_w-\rho_w g\mathrm{z})\\
v_o&=\frac{Kk_{ro}}{\mu_{fo}B_o}(\nabla p_o-\rho_o g\mathrm{z})\\
v_g&=\frac{Kk_{rg}}{\mu_{fg}B_g}(\nabla p_g-\rho_g g\mathrm{z})+R_{so}v_o
\end{aligned} \tag{17}$$

wherev_β, $\beta = w, o, g$ stand for water, oil and gas Darcy velocities, respectively; p_β, $\beta = w, o, g$ stand for water, oil and gas pressures, respectively; ρ_β, $\beta = w, o, g$ denotes for water, oil and gas densities; $k_{r\beta}$, $\beta = w, o, g$ and $\mu_{f\beta}$, $\beta = w, o, g$ relative permeabilities and densities for water, oil and gas. R_{so} is the solution gas-oil ratio.

Saturation equation:

$$S_w+S_o+S_g=1 \tag{18}$$

Capillarity pressure equations defined as functions of saturations:

$$\begin{aligned}
p_{cow}(S_w)&=p_o-p_w\\
p_{cgo}(S_g)&=p_g-p_o
\end{aligned} \tag{19}$$

Initial conditions:

$$\begin{aligned}
p_w&=p_w^0,\quad S_w=S_w^0\\
p_o&=p_0^0,\quad S_o=S_o^0\\
p_g&=p_g^0,\quad S_g=S_g^0
\end{aligned} \tag{20}$$

Boundary conditions:

$$\begin{aligned}
v_w\bullet n&=0\ \ en\ \partial\Omega\\
v_o\bullet n&=0\ \ en\ \partial\Omega\\
v_g\bullet n&=0\ \ en\ \partial\Omega
\end{aligned} \tag{21}$$

We use the following criteria to determine whether the state is three-phase or two-phase:

$\mathrm{N_g} > \mathrm{R_{so}N_o}$, three — phase; $\mathrm{N_g} \le \mathrm{R_{so}N_o}$, two — phase;

Geomechanical Model

The geomechanical model is the same as in the single-phase model, with a change in the gravity term F, since there are now extra variables to consider:

$$\mu\nabla^2 u+(\lambda+\mu)\nabla(\nabla\cdot \mathrm{u})+F-\alpha\nabla \mathrm{p}=0 \tag{22}$$

where

$$F = \rho_s(1-\phi) + \phi\left(\rho_o S_o + \rho_w S_w + \rho_g S_g\right)gz \tag{23}$$

and ρ_s is the solid density.

The Numerical Model for the Single-Phase Case

The standard weigthed residual Galerkin method is used to obtain a numerical model for the fluid flow coupled with geomechanics phenomeno in the single-phase case. The pair of equations (14) and (15), together with the proper initial and boundary conditions, require be expresssed by means a weak or variational formulation, Thomée, V. (2006), in the context of suitable to the problem functional spaces which enjoy enough regularity properties.. The primary variables in the considered problem are the displacement u and the pressure p.

Variational Formulation of the Problem

— $L^2(\Omega)$: Hilbert space of square integrable functions defined in Ω, with interior product and norm defined by

$$(f,g) = \int_\Omega fg\,, \quad [\![f]\!]_{L^2} = (f,f)^{1/2} \tag{24}$$

— $D^{\varsigma}(\Omega)$: partial derivatives of order $\varsigma = (\varsigma_1, \varsigma_2, \ldots, \varsigma_n)$ defined by means

$$D^{|\varsigma|} = \frac{\partial^{|\varsigma|}}{\partial_{x_1}^{\varsigma_1}\partial_{x_2}^{\varsigma_2}\ldots\partial_{x_n}^{\varsigma_n}} \tag{25}$$

with $\varsigma = \varsigma_1, \varsigma_2, \ldots, \varsigma_n$, $n = 2,3$.

— $H^m(\Omega)$: Hilbert space of ordem m, defined as

$$H^m(\Omega) = \{f \in L^2(\Omega) \mid \forall\, |\varsigma| \le m,\ D^{|\varsigma|}f \in L^2(\Omega)\} \tag{26}$$

with norm

$$[\![f]\!]_m = \left(\Sigma_{|\varsigma|\le m}\|f\|_{L^2}^2\right)^{1/2} \tag{27}$$

— $H_0^1(\Omega)$: : Hilbert space of order1, wich reads

$$H_0^1(\Omega) = \{f \in H^1(\Omega) \mid \ f = 0 \ on\ \partial\Omega\ \} \tag{28}$$

— V: function space for displacements, and W: function space for the pressures, defined respectively by

$$\begin{aligned} V &= \{f \in H^1(\Omega) \mid \ f = u_D \ en\ \Gamma_u\ \} \\ W &= \{f \in H^1(\Omega) \mid \ f = p_D \ en\ \Gamma_p\ \} \end{aligned} \tag{29}$$

— $V_h^{r_1} \subset V$ y $W_h^{r_2} \subset W$: space of piecewise continuos polynomial of degree r_1 *and* r_2, respectively.

— $a(u, v)$: bilineal form on v corresponding to the geomechanical equation defined by:

$$a(u,v) = \int_\Omega \{\lambda\,(\nabla\cdot u)(\nabla\cdot v) + 2\mu\,\varepsilon(u):\varepsilon(v)\}d\Omega \tag{30}$$

with $\epsilon(u)$ *y* $\epsilon(v)$ given as in (3), and where the symbol: represents the Frobenius product.

— $b(u, v)$: bilineal form W on corresponding a the fluid flow equation, defined as

$$b(u,v) = \int_\Omega \frac{k}{\eta}(\nabla u\cdot\nabla v)d\Omega \tag{31}$$

The statement for the variational formulation of the above posed strong problema read as following:

Let V and W the spaces defined in (29) *and T the time interval considered. To find {u(x, t), v(x, t)}* $\in$ *VxWxT* such that satisfy the pair of equations

$$\begin{aligned} &\alpha\left(\frac{\partial}{\partial t}(\nabla\cdot u), w\right) + \frac{1}{M}\left(\frac{\partial p}{\partial t}, w\right) + b(p,w) = \hat{q}(w), \quad \forall w \in W \\ &\qquad\qquad and \\ &a(u,v) - \alpha(p, \nabla\cdot v) = \hat{f}(v),\ \forall v \in V \end{aligned} \tag{32}$$

where

$$\hat{q}(w) = \int_{\Gamma_f} \bar{q} w \, d\Omega + (s, w)$$
$$\hat{f}(v) = \int_{\Gamma_t} \bar{t} v \, d\Omega + (f, v) \tag{33}$$

and subject to the initial conditions:

$$p(x,0) = p^0 \; en \; \Omega$$
$$\sigma(x,0) = \sigma^0 \; en \; \Omega \tag{34}$$

Discretization of the Spatial Domain

On the ground of the variational formulation above described, we can carry out a discretization of the special domain, in the following way

Let $V_h^{r_1}, W_h^{r_2}$ *, and T the spaces defined by equation* (29). *To find* $\{u_h(x,t), v_h(x,t)\} \in V_h^{r_1} x W_h^{r_2} x T$ *such that satisfy*

$$\alpha\left(\frac{\partial}{\partial t}(\nabla \cdot u_h), w_h\right) + \frac{1}{M}\left(\frac{\partial p}{\partial t}, w_h\right) + b(p_h, w_h) = \hat{q}(w_h), \; \forall w_h \in W_h^{r_2}$$
$$and$$
$$a(u_h, v_h) - \alpha(p_h, \nabla \cdot v_h) = \hat{f}(v_h), \; \forall v_h \in V_h^{r_1} \tag{35}$$

where

$$\begin{cases} \hat{q}(w_h) = \int_{\Gamma_f} \bar{q} w_h \, d\Omega + (s, w_h) \\ \hat{f}(v_h) = \int_{\Gamma_t} \bar{t} v_h \, d\Omega + (f, v_h) \end{cases} \tag{36}$$

subjet to the initial conditions:

$$\begin{cases} p_h(x,0) = p^0 \; en \; \Omega \\ \sigma_h(x,0) = \sigma^0 \; en \; \Omega \end{cases} \tag{37}$$

In our development and for simplicity, we have chosen $r_1 = r_2 = 1$, that is, equal degree polynomial approximation functions, at the cost of having to face some instability problems in the solutions, Murad, M. and Loula, M., (1994).

Next, we introduce shape functions for the approximated variable u_h and p_h interpolated by the nodos values U and P using the relations:

$$u_h = N_u U$$
$$\varepsilon_h = B_u U$$
$$p_h = N_p P \tag{38}$$

where N_u, B_u *and* N_p are matrices or interpolation vectors for the displacement, deformation and pressure variables, respectively. By inserting these expressions into equations (34) and (35), results a first order differential equation system in terms of the time variable, in the following manner:

$$K\frac{dU}{dt} - L\frac{dP}{dt} - \frac{d\hat{f}}{dt} = 0$$
$$HP + L^T\frac{dU}{dt} + S\frac{dP}{dt} - \hat{g} = 0 \tag{39}$$

with the coefficients given by means of the formulas:

$$K = -\int_\Omega B_u^T D B_u \, d\Omega$$
$$L = \int_\Omega B_u^T D \vec{\alpha} N_p \, d\Omega \tag{40}$$

$$S = \int_\Omega \frac{1}{M} N_p^T N_p \, d\Omega$$
$$H = \int_\Omega \nabla N_p^T \frac{k}{\eta} \nabla N_p \, d\Omega \tag{41}$$

$$\hat{f} = -\int_\Omega N_u^T f \, d\Omega - \int_{\Gamma_t} N_u^T \bar{t} \, d\Omega$$
$$\hat{q} = \int_\Omega N_p^T s \, d\Omega - \int_{\Gamma_p} N_p^T \bar{q} \, d\Omega \tag{42}$$

where $\vec{\alpha} = (\alpha, \alpha, \alpha, 0, 0, 0)$, and D is a constant matrix that express the constitutive stress/strain relationship, written in vectorial notation.

Discretization of the Temporal Domain

The temporal discretization is carried out by using the finite differences method. By denoting

$$X = \begin{bmatrix} U \\ P \end{bmatrix},\ \frac{dX}{dt} = \begin{bmatrix} \frac{dU}{dt} \\ \frac{dP}{dt} \end{bmatrix},\ A = \begin{bmatrix} K & -L \\ 0 & H \end{bmatrix},\ B = \begin{bmatrix} 0 & 0 \\ L^T & S \end{bmatrix},\ F = \begin{bmatrix} \frac{d\hat{f}}{dt} \\ \hat{g} \end{bmatrix} \tag{43}$$

system (38) can be expressed in synthetic form as:

$$B\frac{dX}{dt} + AX = F \tag{44}$$

Equation (43) together with the given initial conditions defines an initial value problem. To solve that initial value problema we will transform equation (43) into an algebraic system by using temporal integration.

We can apply the trapezoidal scheme of two levels, Lewis, R. y Schrefler, B. (1998). The resulting system is:

$$[B + \theta\Delta tA]X_{n+1} = [B - (1-\theta)\Delta tA]X_n + F^{-1}\Delta t \tag{45}$$

That is

$$\begin{bmatrix} \theta K & -\theta L \\ K^T & S + \theta\Delta tH \end{bmatrix}\begin{bmatrix} U_{n+1} \\ P_{n+1} \end{bmatrix} = \begin{bmatrix} (\theta-1)K & -(\theta-1)L \\ L^T & -(1-\theta)\Delta tH \end{bmatrix}\begin{bmatrix} U_n \\ P_n \end{bmatrix} + \begin{bmatrix} \frac{d\hat{f}}{dt} \\ \Delta t\hat{g} \end{bmatrix} \tag{46}$$

where Δt indicates the time increment, U_{n+1}, P_{n+1}, U_n *and* P_n are the desplazaments in the time steps $n+1$ *and* n, respectivenly, while θ is a free parameter that controlls the stability and the accuracy of the scheme. Generally it is imposed $0 < \theta < 1$.

Coupling Strategies

Settari, A. y Walters, D., (2001) identify four possible coupling levels: decoupling, explicit coupling, iterative coupling, and full coupling. In the full coupling strategie the equations are solved simultaneously, and produce the most accurate results but is more computationally demanding.

In the iterative coupling the equations are solved in an independent and iterative way. The coupling between the geomechanical module and the flow fluid module takes place at the end of each time iterationthrough the calcul of the pore volume and porosity changes.

At the end of each time step updated information is sent to the coupling module of the output pressures from the fluid flow module. In turn the geomechanical module send updated information of the porosity changes induced by the pressure and stres changes. The number of iterations is determined by a tolerant parameter in order to get convergency.

Figure 1 summarize the procedure we employ to solve the system, it shows that the system is solved in a sequential way.

Computational Model

The computational model is developed by using the objet oriented C++ programming language, and conducing to the creation of software entities known as “classes” which are usual when using the finite element method..

The main tasks that perform the computational program are the numerical calculations of the coefficients described in equations (39) to (41), the assembling of the element by element matrices associated to the solution of the problem and the resolution of the algebraic equations.

The meshing domain is performed using the open source software Gmesh (hence, with unstructed grids), with tetrahedral elements, while interpolating functions and weighting functions are chosen linear/linear. A typical element is shown in Figure 2.

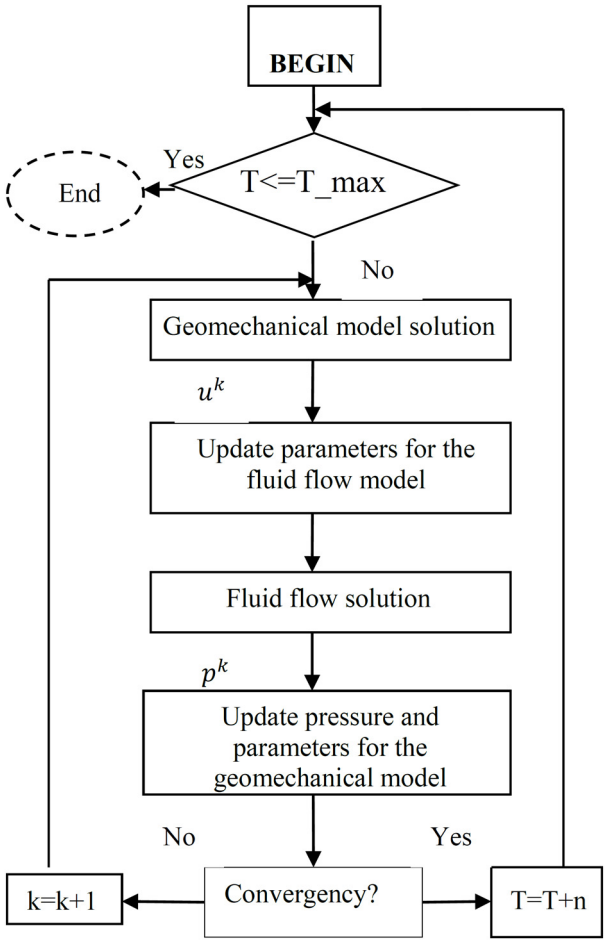

Figure 1—Flow diagram for the fluid flow and geomechanics coupling.

Terzaghi Problem

Consider a 2D porous sample shown in Figure 3, with dimensions 2 m x3 m, under a uniform undrained loading. There are no movement allowed along left and right boundaries, and vertical displacement at its bottom is constrained to zero. Under the undrained conditions fluid is not allowed to flow through the boundaries. The parameters of this sample are listed in Table 1, and were taken from Zheng, Y., R. Burridge, and D. Burns. These authors employ the Young module E and the Poisson ratio ν as elastic constants, but by means a known simple formula we can easily change to the Lamé constants used in our approach.

A mesh consisting of 468 tetrahedrons with a step time of 10 minutes was used in the first simulation, where we have neglected gravity effects. The vertical displacement of the sample top was evaluated during a time interval of 7 hours.

Numerical results of the vertical displacement and pore pressures are exhibited in Figure 4 to 6, along with the analytical solutions, and the results show that the analytical and numerical solutions coincide, which means that the program display an accurately performance for the coupling between the geomechanical and fluid flow models works accurately in this case.

For a second simulation the parameters are listed in Table 2. The sample dimensions and the applied load are the same as in the first simulation, however in this case the gravity effects are not neglected. The simulation was carried out with a 468 tetrahedron numerical mesh with a constant step time of 0.1 minutes.

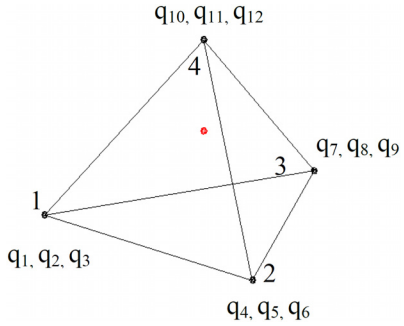

Figure 2—Node enumeration and displacement variables at the local level $x_1=(q_1, q_4, q_7, q_{10})$, $x_2=(q_2, q_5, q_8, q_{11})$, $x_3=(q_3, q_6, q_9, q_{12})$

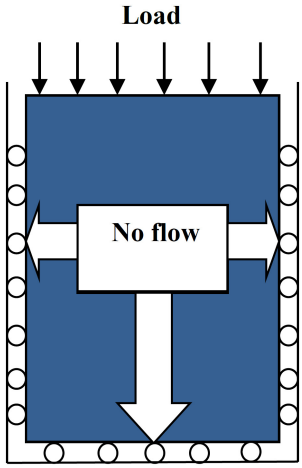

Figure 3—Porous sample in the Terzagui problem

Table 1—Input parameters for first simulation

Young Modulus	1.5E+04 MPa
Poisson Ratio	0.25
Biot Coefficient	0.85
Dinamic Viscosity	5E-07 MPa.s
Permeability	5E-14 m^2
Total Compressibility	7.69E-05(1/MPa)

The vertical displacements profiles were evaluated during a 1 minute and a 10 minutes time intervals, allowing us to compare our results with of the Zheng et al. Figure 7 to 8 below show a comparisson between both results, showing very close results of the displayed profiles from the program and from the cited authors, which means that the program also works correctly with activated gravity.

Conclusions

A mathematical model that accounts for the fluid flow in a porous medium in the single and multiphase case and its interaction with the gomechanical changes have been developed. The strong formulation of the problem has been provided by means an initial and boundary value problem with pressure and displacement as primary variables in the single phase case, and with water pressure, water concentration and gas concentrarion in the multiphase case among several options.

A numerical model is provided for the single phase case by using a variational approach founded in the weighted Galerking finite element method for both fluid flow model and geomechanics model. The trial and shape functions are chosen as linear polynomials. The spatial discretization leads to a differential equation with respect to time, which can be solved by means of the finite difference method.

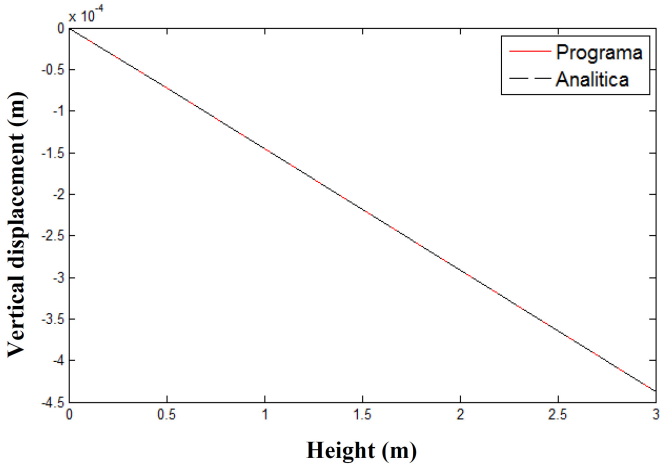

Figure 4—Vertical displacement after load application, numerical and analytical comparison under undrained loading.

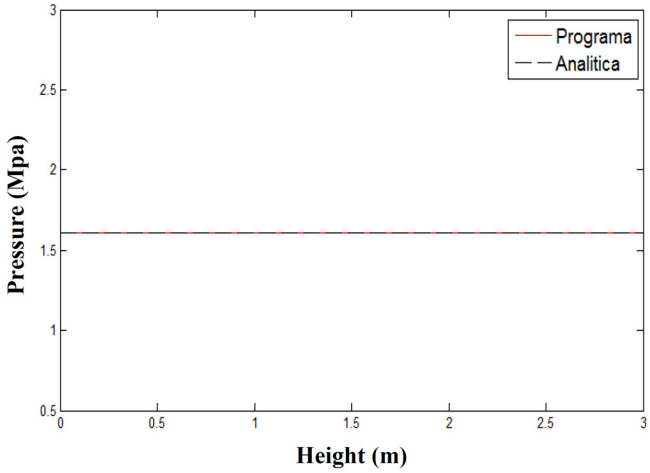

Figure 5—Uniform pore pressure inside the sample, numerical and analytical comparison under undrained loadig

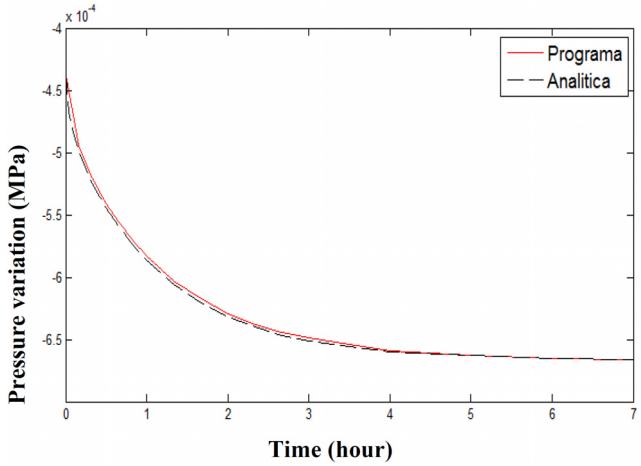

Figure 6—Pore pressure decline, numerical and analytical comparison under undrained loading during 7 hours

The computational model has been approached in the context of C++ language and has been implemented in the single phase case. The main tasks that perform the computational program are the numerical calculations of the coefficients described in the variational formulation, the assembling of the element by element matrices associated to the solution of the problem and the resolution of the algebraic

Table 2—Input parameters for simulation 2

Young Modulus	1.44E+04 MPa
Poisson Ratio	0.2
Biot Coefficient	0.79
Total Compressibility	8.13E-05 (1/MPa)
Rock Density	2000Kg/m^3
Rock Density	940kg/m^3
Porosity	0.2
Permeability	2E-13m^2
Cinematic Viscosity	1.3E-4 m^2/s

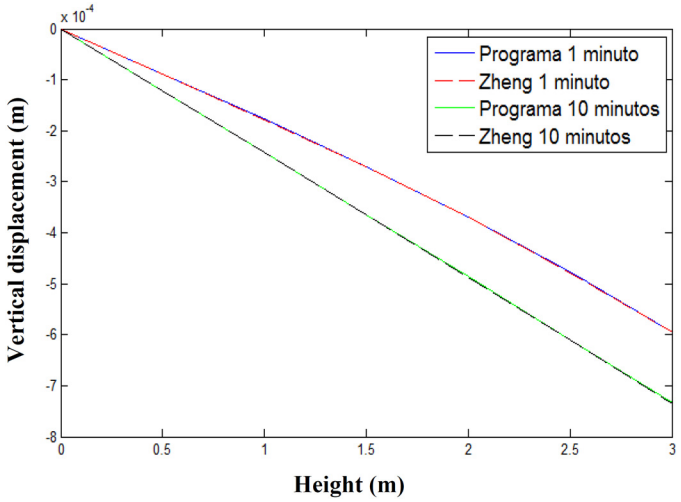

Figure 7—Displacement profiles of the sample, comparison with Zheng et al solution, 1 minute, and 10 minutes time intervals

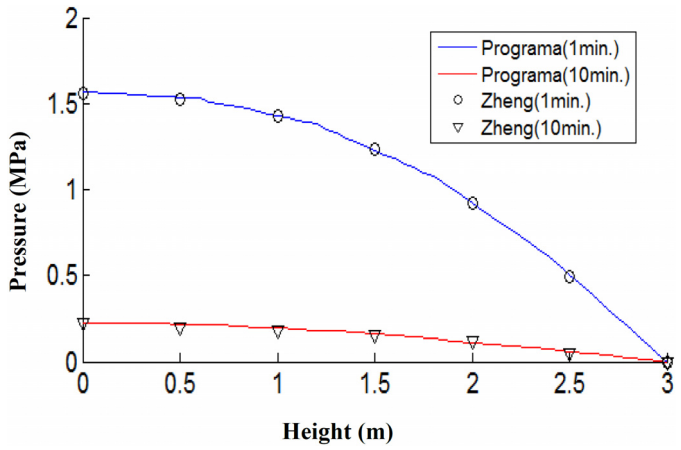

Figure 8—Pressure behavior, comparison with Zhen et al solutions during two time intervals

equations. The development of this code is still in its initial stage, but the approach show promise, as shown by the numerical solution of the Terzaghi problem.

As a future work it would be convenient to extend the obtained results to more general situations that includes analysis of geomechanical effects on permeability, non isothermal processes, behavior of the rock beyond the simplest linear elastic case such as elastoplastic behavior, and to study natural fractured or unconventional reservoirs.

Another resarch line that deserves attention is related with the application of more stable versions of the standard Galerkin method, or non standard versions such as mixed or discontinuous methods, or alternative methods such as volume finite element.

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Nomenclature

$a(u, v)$	: bilineal formon VxV
$b(u, v)$	: bilineal formon WxW
B_u	: deformation expression
B_u	: volume factor for the phase β
$D^{\|s\|} f$	: partialderivatives of$f$ of orderς
E	: Young modulus
F	: force vector on the body
(f, g)	: interior producton $L^2(\omega)$
g	: gas phase
$H_0^1(\Omega) = \{f \in H^1(\Omega) \mid f = 0 \, en \, \partial\Omega \}$	: Hilbert space of order 1
$H^m(\Omega)$	: Hilbert space of order$m$
K_f, K_s	: fluid bulk modulusand drained skeleton bulk modulus
K	: permeability tensor
$k_{r\beta}$	: relative permeability of the phaseβ
$L^2(\beta)$	: Hilbert space of square integrable functions onΩ
M	: Biot modulus
N_β	: concentration of the phase β
N_p	: pressure shape functions
N_u	: displacement shape functions
o	: oil phase
$p_{cow}(S_w)$	: oil-water capillary pressure
$p_{cgo}(S_g)$	: gas-oil capillary pressure
p	: pore pressure
$p\beta$	: pore pressure of the phase β
$q\beta$	: source/sink term of the phaseβ
R_{so}	: dissolved-gas/oil ratio
S_β	: saturation of tha phase β
u	: displacement vector
V	: displacement functional space
$V_h^{r_1}, W_h^{r_2}$	: spaces of piecewise continuous polynomial functions of degree r_1 andr_2
$\overline{v_\beta}$:	: flow velocity of the phase β
v_s	: phase solid velocity
W	: pressures functional space
w	: water phase
α	: Biot coefficient
ϵ	: deformation tensor
ϵ_v	: volumetric deformation

σ	: stress tensor
σ'	: effective stress tensor
Ω, $\partial\Omega$	: domain of interest and its boundary
λ, μ	: Lamé elastics constants
$\mu\beta$	: viscosity of the fluid phase β
η	: increment of fluid content
ϕ	: material porosity
ϕ^*	: relative porosity
v	: razón de Poisson

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